12/21/16

"Some are born to greatness, some achieve greatness, and some have greatness thrust upon them"
- William Shakespeare

HW: "Circles HW" #1-6

AIM: How do we write the equation of a circle?

Warm Up: ✱ $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

What is the distance between the points (4, 2) and (7, -2)? (Simplest Radical Form)

x_1, y_1 x_2, y_2

$d = \sqrt{(4-7)^2 + (2--2)^2}$

$d = \sqrt{9 + 16}$

$d = \sqrt{25}$

$d = 5$

THE EQUATION OF A CIRCLE

A circle whose center is at (h, k) and whose radius is r is given by: $(x-h)^2+(y-k)^2=r^2$

***Exercise* #2:** Which of the following equations would have a center of $(-3, 6)$ and a radius of 3?

(1) $(x-3)^2+(y+6)^2=9$ — 3, -6

(2) $(x+3)^2+(y-6)^2=9$

(3) $(x-3)^2+(y-6)^2=3$ — error

(4) $(x+3)^2+(y+6)^2=3$

$3^2=9$

***Exercise* #3:** For each of the following equations of circles, determine both the circle's center and its radius. If its radius is not an integer, express it in decimal form rounded to the nearest *tenth*.

(a) $(x-2)^2+(y-7)^2=100$

C: (2, 7)

r: $\sqrt{100}=10$

(b) $(x-5)^2+(y+8)^2=4$

C: (5, -8)

r: $\sqrt{4}=2$

(c) $x^2+y^2=121$ — $(x+0)^2+(y+0)^2=121$

C: (0, 0)

r: $\sqrt{121}=11$

(d) $(x+1)^2+(y+2)^2=1$

C: (-1, -2)

r: $\sqrt{1}=1$

(e) $x^2+(y-3)^2=49$

C: (0, 3)

r: 7

(f) $(x+6)^2+(y-5)^2=18$

C: (-6, 5)

r: $\sqrt{18}=3\sqrt{2}$

r: 4.2

***Exercise* #5:** By completing the square on both quadratic expressions in x and y determine the center and radius of a circle whose equation is

STANDARD FORM

$$x^2+10x+y^2-2y=10$$

$$x^2+10x+\boxed{25}+y^2-2y+\boxed{1}=10+\boxed{25}+\boxed{1}$$

$\frac{10}{2}=5 \quad 5^2=25$ $\quad -\frac{2}{2}=-1 \quad (-1)^2=1$ $\quad 36$

Center/Radius Form

$$(x+5)^2+(y-1)^2=36$$

center: $(-5,1)$

radius: $\sqrt{36}=6$

HW) 5)

A) C(5,2) A(11,10)

* Radius = the distance from the center (c) to any point on the circle

radius $= \sqrt{(11-5)^2 + (10-2)^2}$

$= \sqrt{36+64}$

$= \sqrt{100}$

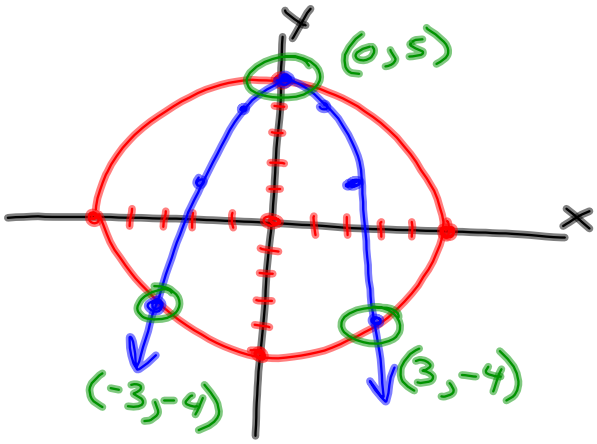
radius = 10

$(x-5)^2 + (y-2)^2 = 100$

6) $x^2 + y^2 = 25$ ← circle

c: (0,0)

r: 5



$y = 5 - x^2$ ← parabola

calculator

x	y
-3	-4
-2	1
-1	4
0	5
1	4
2	1
3	-4

Warm Up:

$$x^2 + y^2 + 14x + 12y + 76 = 0$$

$$x^2 + 14x + \boxed{49} + y^2 + 12y + \boxed{36} = -76 + \boxed{49} + \boxed{36}$$

$$9$$

$\frac{14}{2} = 7$

$7^2 = 49$

$$(x+7)^2 + (y+6)^2 = 9$$

$\frac{12}{2} = 6$

$6^2 = 36$

Center: $(-7, -6)$

Radius: $\sqrt{9} = 3$

1) $4x^2 + 4y^2 - 16x - 24y + 51 = 0$

$\frac{4x^2}{4} - \frac{16x}{4} + \square + \frac{4y^2}{4} - \frac{24y}{4} + \square = \frac{-51}{4} + \square + \square$

$x^2 - 4x + \boxed{4} + y^2 - 6y + \boxed{9} = \frac{-51}{4} + \boxed{4} + \boxed{9}$

$\frac{-4}{2} = -2$, $(-2)^2 = 4$

$\frac{-6}{2} = -3$, $(-3)^2 = 9$

$\frac{1}{4}$

$(x-2)^2 + (y-3)^2 = \frac{1}{4}$

Center: $(2, 3)$

Radius: $\sqrt{\frac{1}{4}} = \frac{1}{2}$

7) $x^2 + y^2 + 3y = \frac{1}{4}$

* Make sure the coefficient of x^2 and y^2 is 1.

$x^2 + y^2 + 3y + \boxed{\frac{9}{4}} = \frac{1}{4} + \boxed{\frac{9}{4}}$

$\frac{10}{4} = \frac{5}{2}$

$\frac{3}{2}$ $\left(\frac{3}{2}\right)^2$

$x^2 + \left(y + \frac{3}{2}\right)^2 = \frac{5}{2}$

center: $\left(0, -\frac{3}{2}\right)$

Radius: $\sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$

14) Center: (12, 6)

Area = 49π $\rightarrow$ $49 = r^2$

$\boxed{7 = r}$

* Recall: Area = πr^2

$(x-12)^2 + (y-6)^2 = 7^2$

$(x-12)(x-12) + (y-6)(y-6) = 49$

$x^2 - 12x - 12x + 144$ $y^2 - 6y - 6y + 36$

$x^2 - 24x + 144 + y^2 - 12y + 36 = 49$

$-49 \quad -49$

$\boxed{x^2 - 24x + y^2 - 12y + 131 = 0}$

$x^2 + y^2 - 24x - 12y + 131 = 0$